

## ORIGINAL RESEARCH ARTICLE

## The Folding Potential of Elastically Scattered $d + {}^{24}\text{Mg}$ Employing B3Y-Fetal Interaction within the Double Folding Model Framework

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## ARTICLE HISTORY

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## ABSTRACT

The analysis of the deuteron scattering from  ${}^{24}\text{Mg}$  was performed in the elastic channel using the double-folding model to evaluate the optical potentials of the present study. Both the real and the imaginary parts of the optical potentials were computed using a mass-dependent interaction (B3Y-Fetal) in the double-folding formalism. The derived double-folding potentials were used to analyse the differential cross-sections of  $d + {}^{24}\text{Mg}$  within the energy range of 60–170 MeV. With the derived optical potentials, the reaction and differential cross-sections of  $d + {}^{24}\text{Mg}$  were extracted in the optical model. The plots of the computed differential cross-sections were made and the results were compared to those obtained experimentally to ascertain the suitability of the derived potentials and the B3Y-Fetal interaction. In conclusion, the optical potentials obtained using the double-folding model with the B3Y-Fetal interaction were successful in reproducing the experimental data.

## KEYWORDS

B3Y-Fetal, Cross-Section, Double-folding, Elastic Channel, Folding potential

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## INTRODUCTION

In recent years, nuclear physicists have paid a great deal of attention to the topic of developing the most appropriate type of nuclear potential to study nuclear interactions (Kurkuoglu et al., 2006). Over the years, several theoretical models that explain nuclear interactions and the structure of the interacting nuclei have been developed to compute nuclear potential (Ibraheem et al., 2023). The application of computational techniques in determining nuclear optical potentials has resulted in unprecedented success. To this end, phenomenological and folding potentials were developed and widely used in nuclear structure and reaction studies (Amer et al., 2016).

Over the years, the densities of the interacting nuclei have been linked with actual nucleon-nucleon (NN) interactions within the framework of the folding model (FM) to provide a good description of the nuclear optical potentials (Farid et al., 2001). The ability to employ folding potentials to give a coherent description of interacting systems makes them interesting. The phenomenological potentials' complexity is decreased with folding potentials (Farid, 2002). The target and arriving nuclei's density distributions are integrated with an effective nucleon-

nucleon interaction to compute the folding potentials (Hagino et al., 2006).

The optical potentials derived from the folding models are considered suitable due to their ease of manipulation and the ability of the folding formalism to provide a comprehensive description of the interacting system. By utilising a phenomenological approach, the complex optical potential's imaginary component is considered, allowing for microscopic analysis of the folded potentials while taking into consideration the effects of absorption. The application of folding potentials in the analysis of scattering data in the elastic channel for nuclear reaction investigations has garnered significant interest (Behairy et al., 2015; Farid et al., 2014; Satchler, 1983).

In this work, rather than using phenomenological potentials to account for the imaginary part of the optical potential, the double-folding model was used to evaluate both the real and imaginary parts of the optical potentials for the optical model analysis. The suitability of the B3Y-Fetal interaction for nuclear reaction studies was also taken into consideration. Hence, the double-folding model is employed in this study to ascertain the intricate optical potential. The objective of this work is to develop an optical potential to examine the scattering data of the

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reaction between a deuteron ( $d$ ) and a  $^{24}\text{Mg}$  nucleus in the elastic channel.

## THEORETICAL FRAMEWORK

Different forms of nucleon-nucleon interactions, such as the Michigan three Yukawa interaction (M3Y) interaction, have been used for the studies of nuclear matter properties (Anantaraman et al., 1983; Burtebayev et al., 2020; Kobos et al., 1982; Satchler & Love, 1979). The M3Y interactions were used to study nuclear matter properties and reactions and were quite successful. In a recent study, mass-dependent interactions dubbed the B3Y-Fetal interaction were derived. The interactions were derived using mass numbers  $A = 16, 24, 40,$  and  $90$ . Although the interactions are systematically mass-dependent, it is recommended that they be used in the study of nuclear structure and reactions for all nuclei. (Fiase et al., 2002).

In this study, a mass-dependent interaction dubbed the B3Y-Fetal interaction was derived using the strengths of the isoscalar component of the central potential of Table VII (Fiase et al., 2002). The direct,  $V_{00}^D(s)$  and exchange,  $V_{00}^{Ex}(s)$  components of the isoscalar B3Y-Fetal interaction may be expressed as (Abenga et al., 2020),

$$V_{00}^D(s) = 7419.23 \frac{e^{-4s}}{4s} - 1823.98 \frac{e^{-2.5s}}{2.5s}, \quad (1)$$

similarly, the exchange term is determined as follows:

$$V_{00}^{Ex}(s) = 4745.02 \frac{e^{-4s}}{4s} - 1984.144 \frac{e^{-2.5s}}{2.5s} - 7.8474 \frac{e^{-0.7072s}}{0.7072s} \quad (2)$$

In order to incorporate the impact of antisymmetrization resulting from the exchange of nucleons between the projectile and target nuclei, the folding calculations include the contribution of single nucleon knock-on exchange. Research has demonstrated that the effects of knock-on exchange may be precisely calculated by using a zero-range pseudo-potential,  $\mathcal{F}_{Ex}\delta(r)$ , (Brandan & Satchler, 1997; Love & Owen, 1975), and the interaction for the folding calculation becomes,

$$V_{tp}(s, E) = \left[ 7419.23 \frac{e^{-4s}}{4s} - 1823.98 \frac{e^{-2.5s}}{2.5s} \right] + \mathcal{F}_{Ex}(E)\delta(s), \quad (3)$$

The inclusion of the zero-range pseudo-potential is essential to incorporate the exchange component within the effective interaction. The form of the zero-range pseudo-potential included in equation (6) is given as,

$$\mathcal{F}_{Ex} = J_{Ex}g(E) \quad (4)$$

where  $g(E)$  is an energy scaling factor. According to the calculations by (Abenga et al., 2020, 2021), the exchange amplitude  $J_{Ex}$  of the B3Y-Fetal interaction has a magnitude of around  $-361$  MeV. The energy scaling factor of the B3Y-Fetal contact was estimated to be  $\left[ 1 - 0.005 \left( \frac{E}{A} \right) \right]$ , which is similar to that of the M3Y-Paris

interaction (Khoa et al., 1996) in order to accommodate the energy variance.

Studies have shown that the nuclear matter properties are not reproduced by the bare NN (B3Y-Fetal) interaction alone. Therefore, a density-dependence component had to be added to the original B3Y-fetal interaction (Khoa & Von Oertzen, 1993). The primary goal of adding the density dependence factor to the bare interaction is to stabilise the nuclear matter rather than cause it to collapse at the saturation condition (Zang et al., 2007). The density-dependent component  $f(\rho)$  is included into the effective interaction according to the proposition given by (Hamada et al., 2016; Moharram & El-Shal, 2002),

$$V_{00}^{DD}(s, \rho, E) = f(\rho)V_{tp}(s, E) \quad (5)$$

The explicit form of the density dependence form factor under consideration is the exponential one, and it is expressed as (Khoa et al., 1996; Khoa & Von Oertzen, 1993),

$$f(\rho) = C(1 + \alpha \exp(-\beta\rho)) \quad (6)$$

where  $C, \alpha,$  and  $\beta$  are constants. Equation (3) was parametrized by (Abenga et al., 2020) at the saturation state to yield the accurate value of the nuclear matter binding energy at  $\epsilon = -16$  MeV.

## OPTICAL MODEL: DOUBLE-FOLDING POTENTIAL

The optical potential,  $U(r)$ , that was employed in this investigation was thought to be complex. The potential strengths are calculated using the double-folding model approach. In the folding formalism, the folding potential,  $V_F(r)$ , was computed by averaging the density-dependent B3Y-Fetal interaction and the densities of the interacting nuclei (projectile  $\rho_p$  and target  $\rho_t$ ). The potential of the double-folding model is provided by (Brandan & Satchler, 1997; Satchler & Love, 1979);

$$V_F(\mathbf{r}) = \iint \rho_p(\mathbf{r}_p)\rho_t(\mathbf{r}_t)V_{pt}(\mathbf{s} - \mathbf{r}_p + \mathbf{r}_t)d^3r_p d^3r_t, \quad (7)$$

where  $V_{pt}$  is the density-dependent B3Y-Fetal interaction defined by Equation (5).

The Fermi-type function was used in this investigation to characterise the target or incoming nuclei's nuclear and charge densities. Specifically, the two-parameter Fermi (2pF) function, which is a Fermi function with two parameters, was applied. The 2pF function is defined as (El-Attar et al., 2008),

$$\rho_{n(p)}(r) = \rho_{0n(p)} \left[ 1 + \exp\left(\frac{r - R_{p(n)}}{a_{p(n)}}\right) \right]^{-1} \quad (8)$$

where  $\rho_0, R_{p(n)},$  and  $a_{p(n)}$  are the nuclear matter density parameters.

The proton (neutron) half-density radii  $R_{p(n)}$  were parameterised as (De Vries et al., 1987; Olorunfunmi & Olatinwo, 2023),

$$\left. \begin{aligned} R_p &= 0.449 + 0.0071 \left( \frac{Z}{N} \right) \\ R_n &= 0.446 + 0.0072 \left( \frac{N}{Z} \right) \end{aligned} \right\} \quad (9)$$

The numerical value of  $\rho_0$  is determined from the condition that  $\rho_0 = Z$  for proton and  $\rho_0 = A - Z$  for neutron (De Vries et al., 1987).

The effective interaction included in Equation (7) is real, and it is expected to account for the bare potential only. The immediate consequence was that the imaginary potential, which accounts for the polarization effect, must be constructed separately. However, this work accounted for the polarization effect by including an imaginary potential using the folding approach. With this proposition, the present nuclear potential was taken in the form (Abenga et al., 2023):

$$U(r) = (N_r + iN_i)V_F(r) + V_{Coul}(r) \quad (10)$$

Here,  $N_{r(i)}$  represents the potential adjustment parameters for the complex nuclear potential, whereas  $V_{Coul}(r)$  denotes the Coulomb potential. The use of the

renormalization parameters was to enhance the accuracy of the theoretical calculations to align with the observed data.

## RESULTS

The angular distribution data of elastically scattered  $d + {}^{24}\text{Mg}$  within the incident laboratory, energies of 60 – 170 MeV were analysed utilising the DFM approach. The components of the complex nuclear potentials were computed employing the optical model framework outlined in Section 3. The optical potential was characterised by both the real and imaginary components, which were assumed to be equivalent but adjusted using distinct renormalization factors.

The calculated reaction ( $\sigma_r$ ) cross-section is detailed in Table 1. In comparison to results derived from phenomenological and semi-microscopic potentials (Bäumer et al., 2001; Farid et al., 2014; Ibraheem, 2016), the current computation reveals reduced values of  $\sigma_r$ . The parameters of optimum fit from the folding calculations and the accompanying evaluated reaction cross-sections are shown in Table 1.

**Table 1:** Best-fit parameters deduced from the double folding calculations and other kinematical parameters of elastic scattering of  $d + {}^{24}\text{Mg}$ .

Target	$E_{lab}$ (MeV)	$N_r$	$N_i$	$\sigma_r$ (mb)
${}^{24}\text{Mg}$	60.0	1.30	0.60	1048.35
	60.6	1.25	0.70	1077.21
	64.0	1.20	0.60	1021.51
	66.0	1.20	0.60	1013.40
	68.0	1.20	0.60	1005.45
	70.0	1.18	0.60	995.85
	72.0	1.20	0.60	989.99
	74.0	1.20	0.65	1002.96
	76.0	1.20	0.65	995.74
	77.3	1.20	0.65	991.11
	78.0	1.20	0.65	988.63
	80.0	1.15	0.58	947.67
	90.0	1.18	0.6	925.06
	170.0	1.20	0.75	770.26

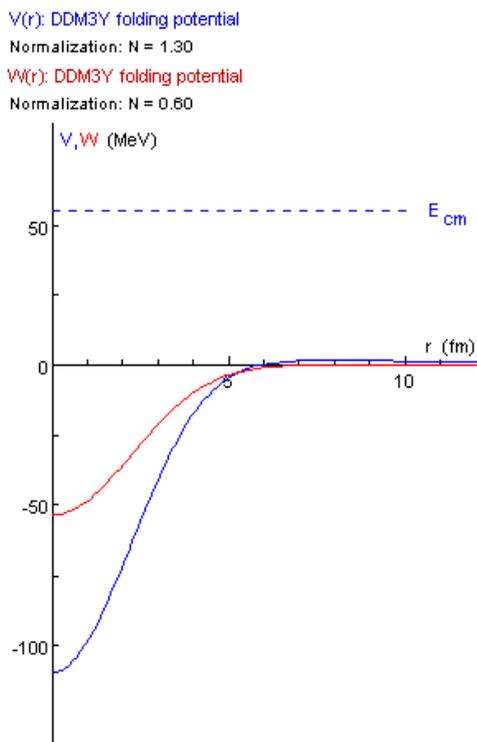


Figure 1: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 60$  MeV.

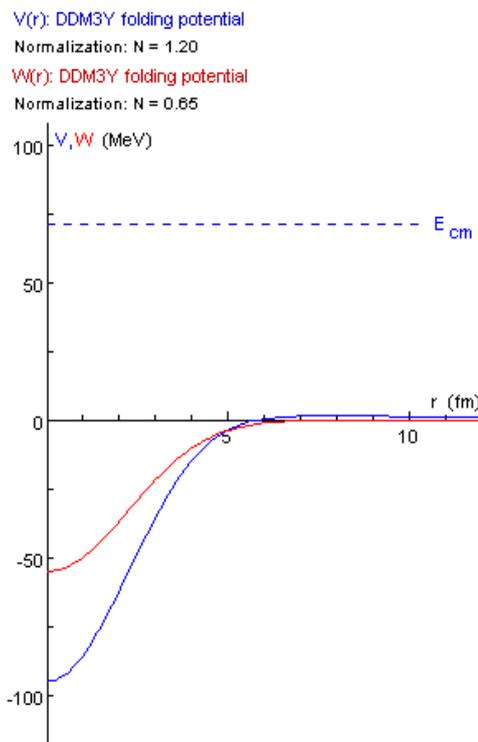


Figure 2: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 77.3$  MeV.

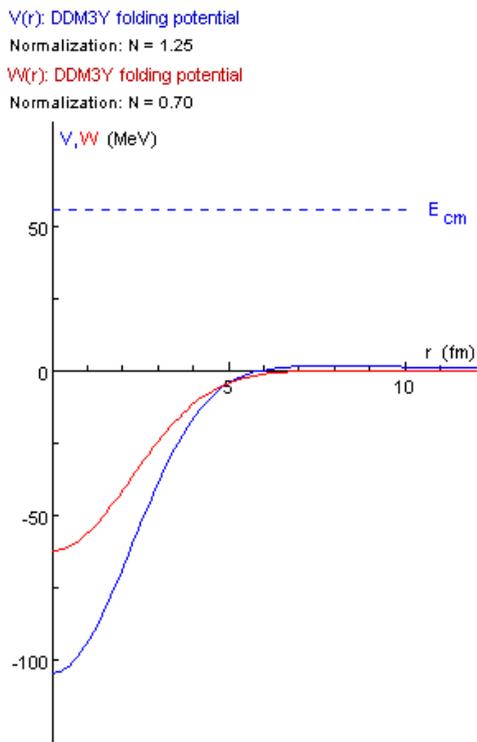


Figure 3: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 60.6$  MeV.

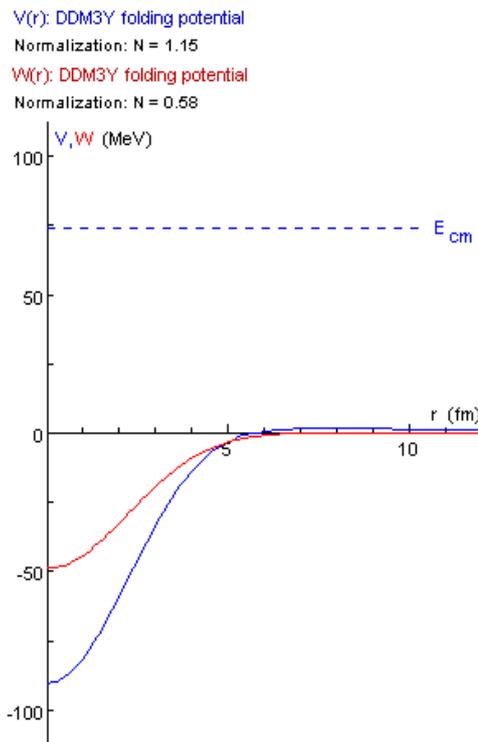


Figure 4: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 64$  MeV.

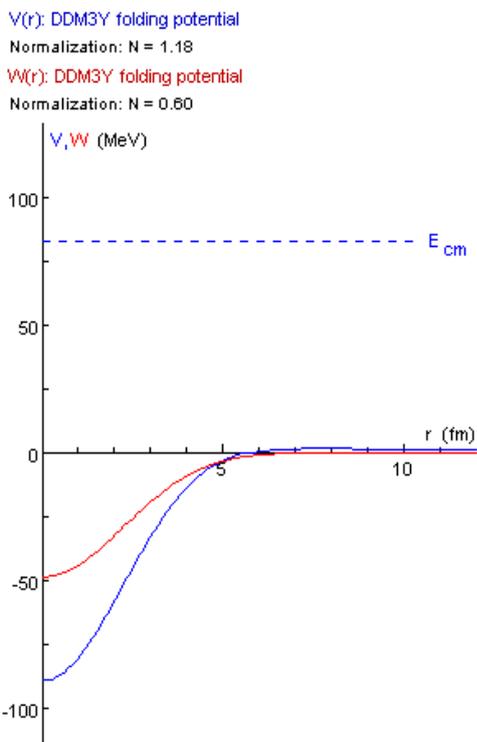


Figure 5: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 66$  MeV.

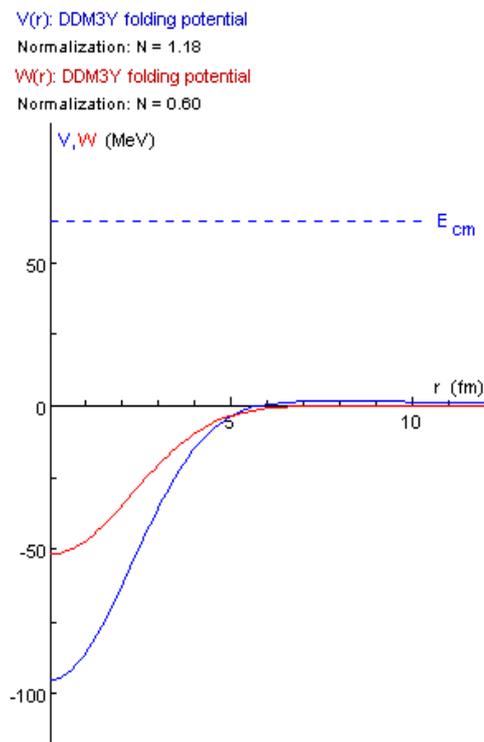


Figure 6: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 70$  MeV.

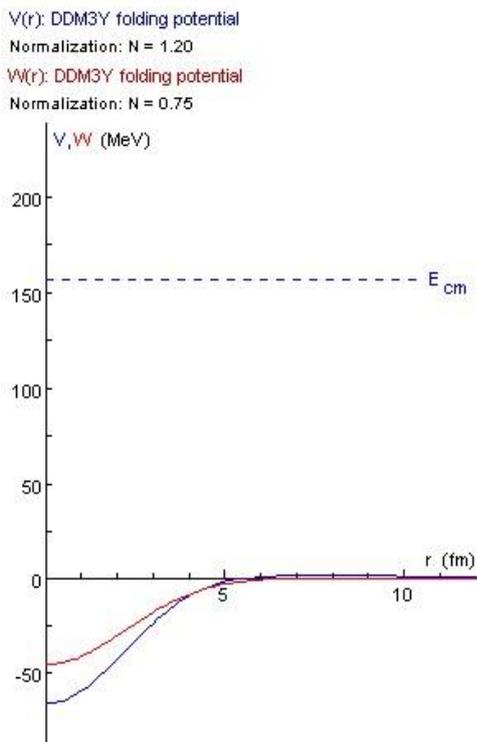


Figure 7: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 68$  MeV.

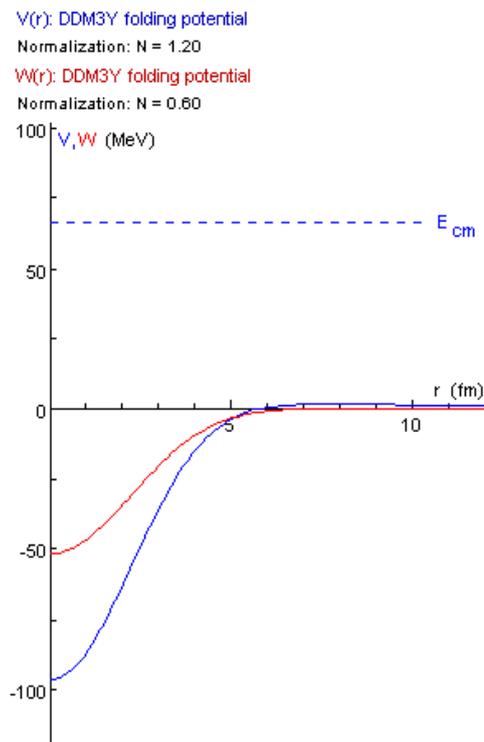


Figure 8: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 72$  MeV.

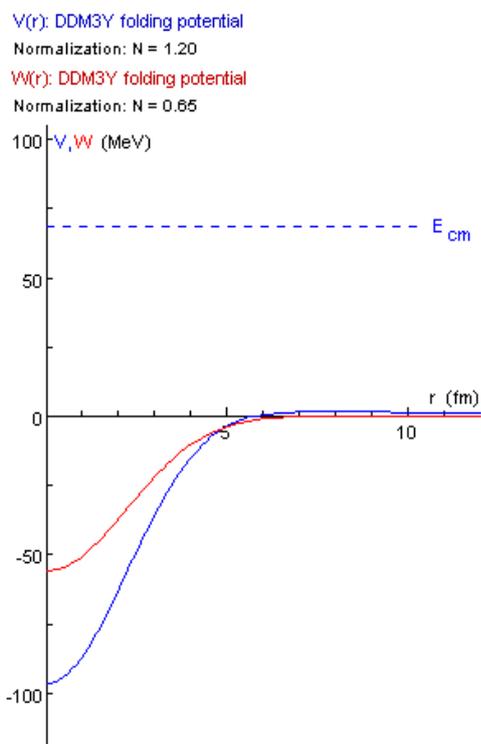


Figure 9: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 74$  MeV.

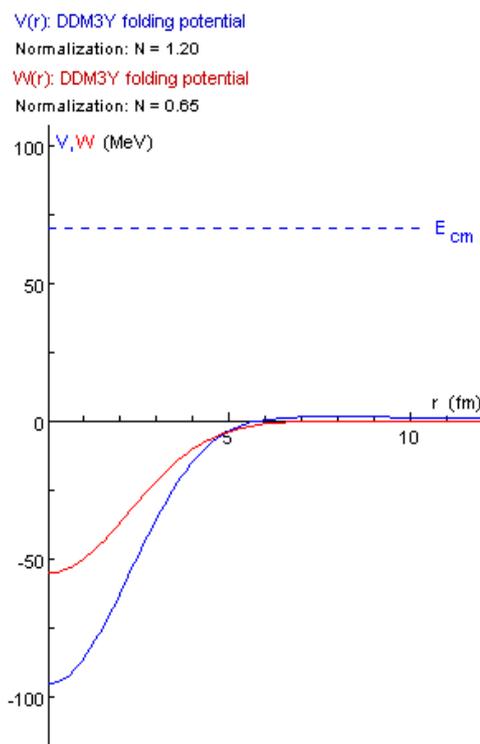


Figure 10: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 76$  MeV.

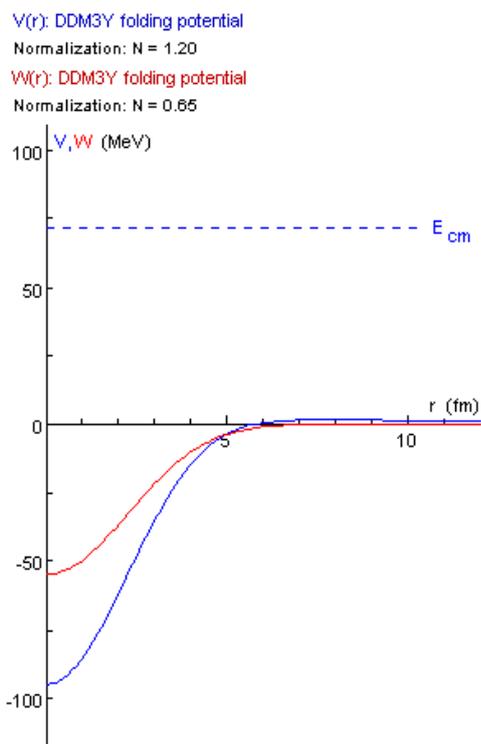


Figure 11: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 78$  MeV.

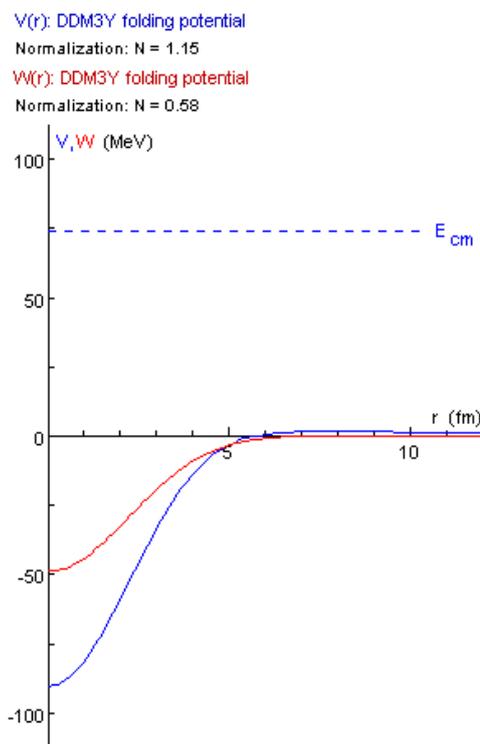


Figure 12: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 80$  MeV.

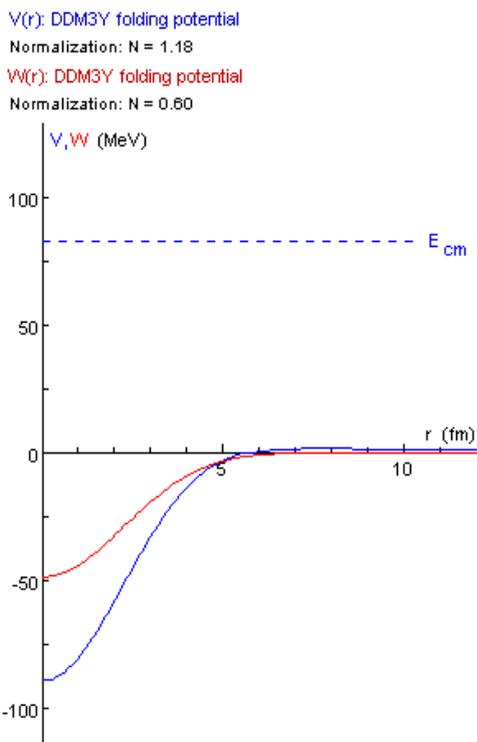


Figure 13: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 90$  MeV.

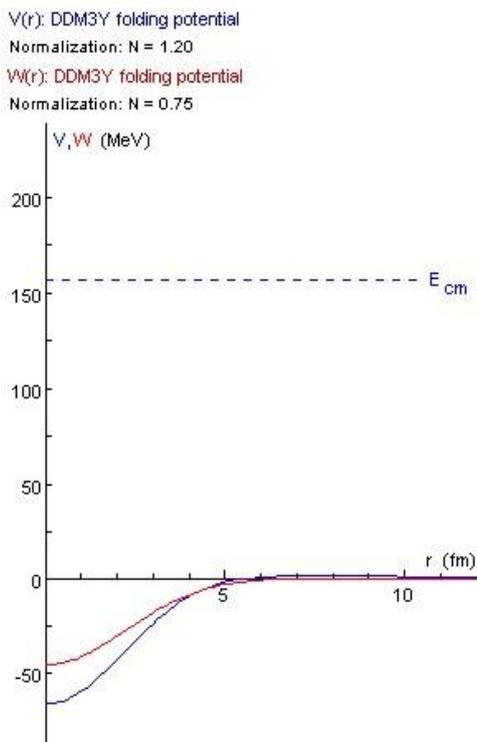


Figure 14: Optical potential of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 170$  MeV.

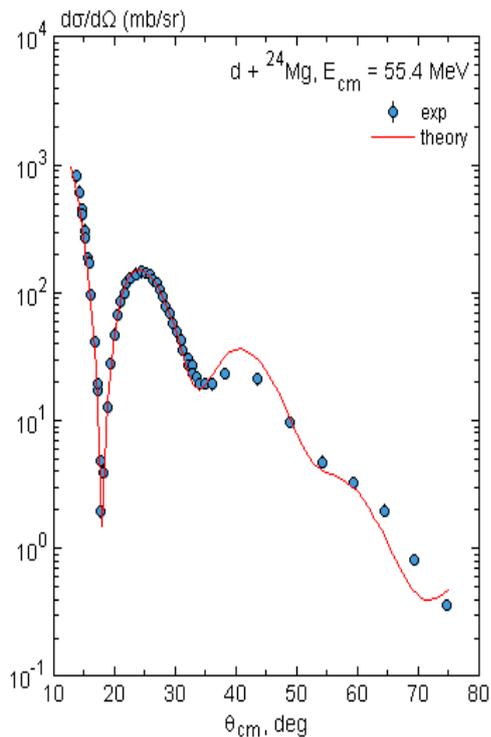


Figure 15: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 60$  MeV.

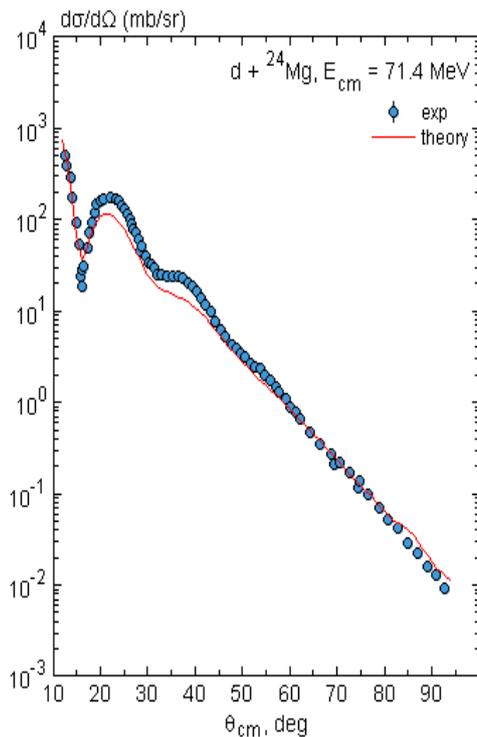


Figure 16: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 62$  MeV.

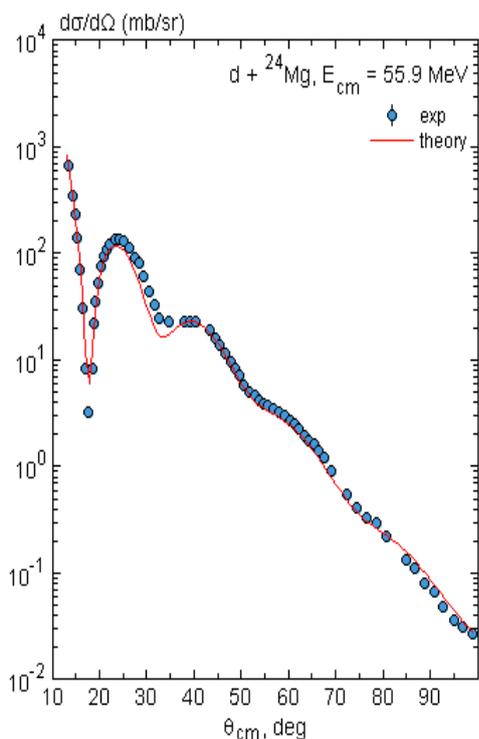


Figure 17: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 60.6$  MeV.

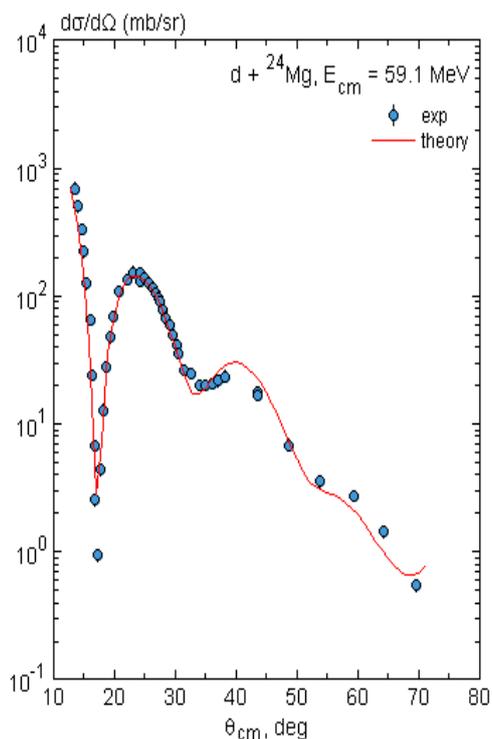


Figure 18: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 64$  MeV.

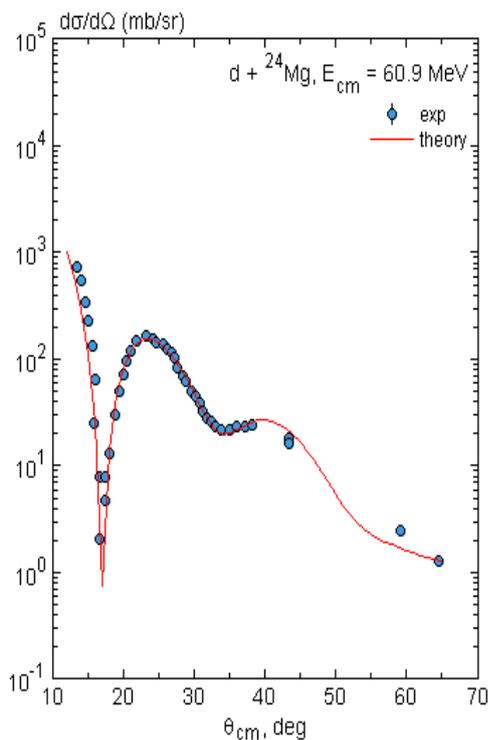


Figure 19: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 66$  MeV.

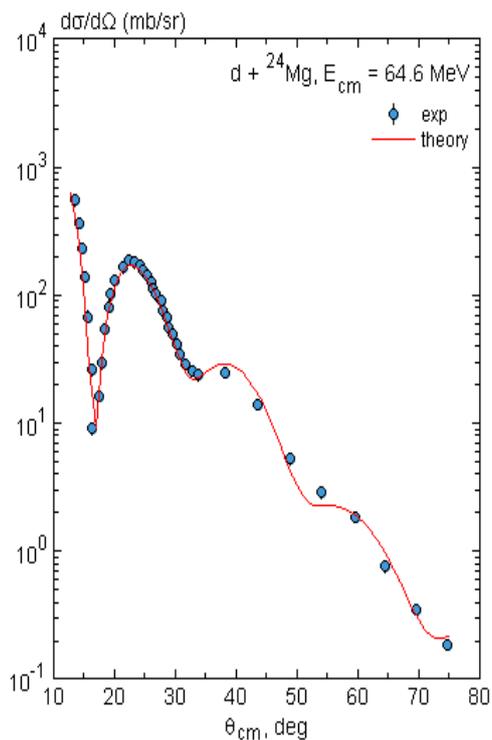


Figure 20: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 70$  MeV.

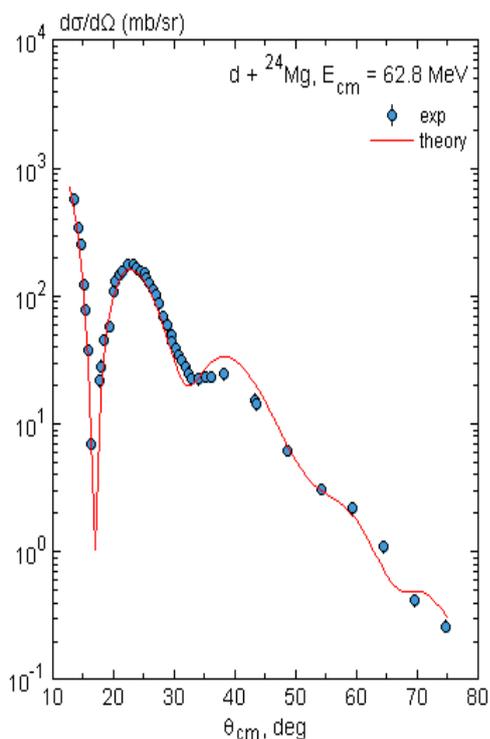


Figure 21: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 68 \text{ MeV}$ .

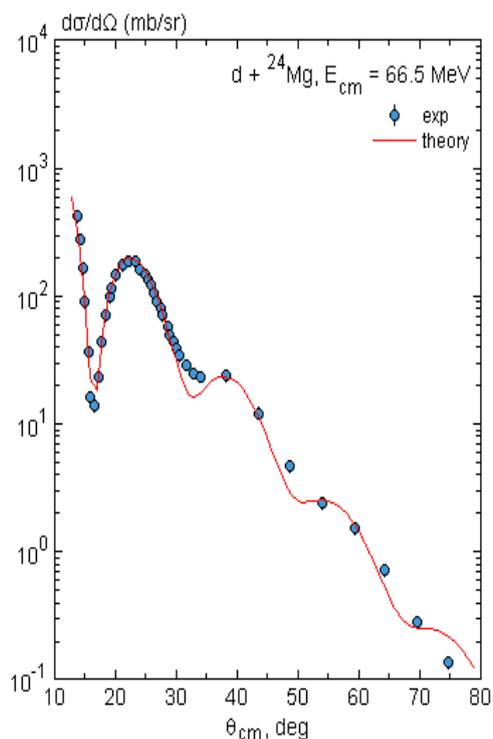


Figure 22: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 72 \text{ MeV}$ .

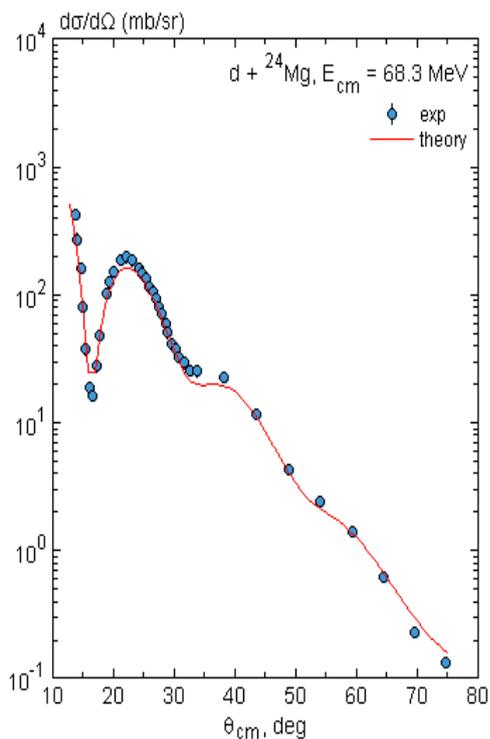


Figure 23: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 74 \text{ MeV}$ .

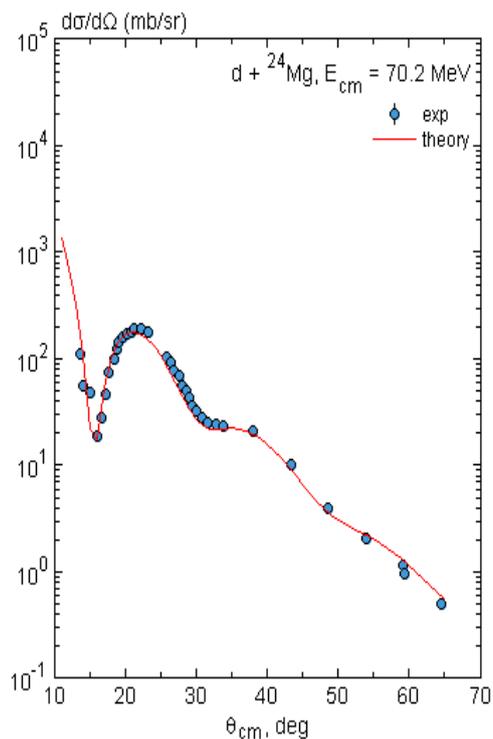


Figure 24: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 76 \text{ MeV}$ .

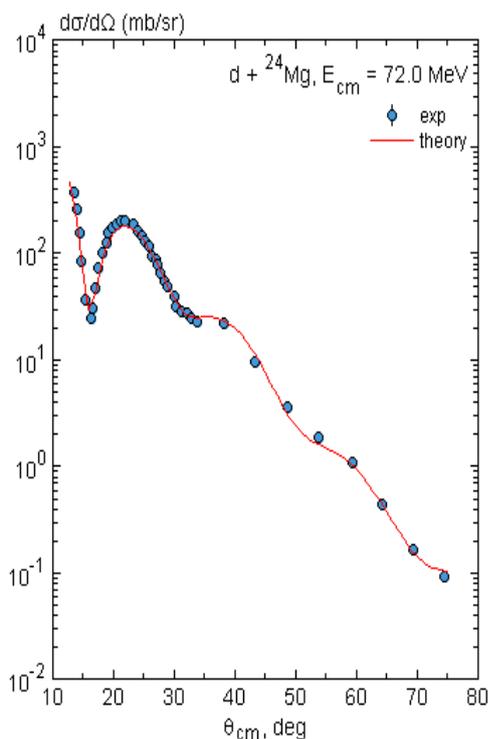


Figure 25: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 78 \text{ MeV}$ .

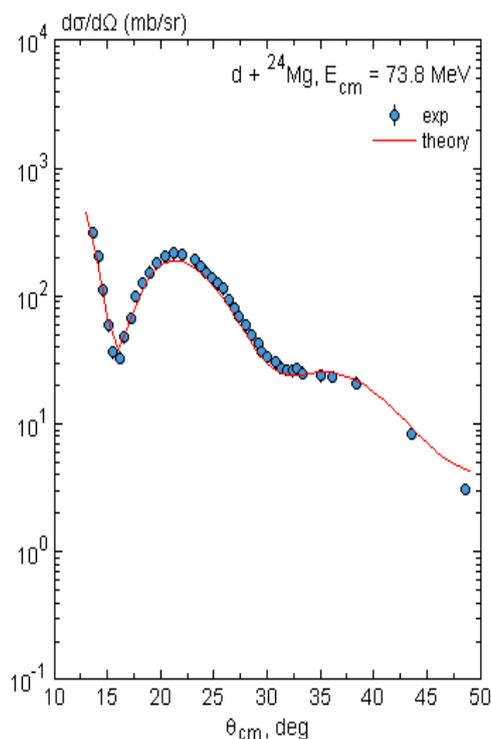


Figure 26: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 80 \text{ MeV}$ .

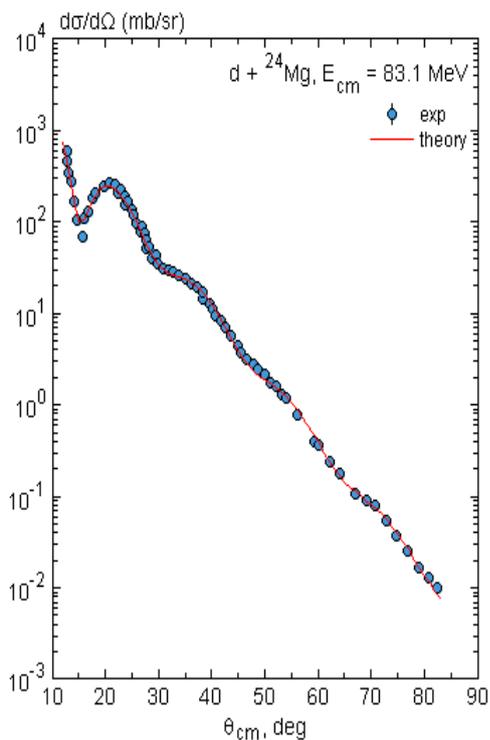


Figure 27: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 90 \text{ MeV}$ .

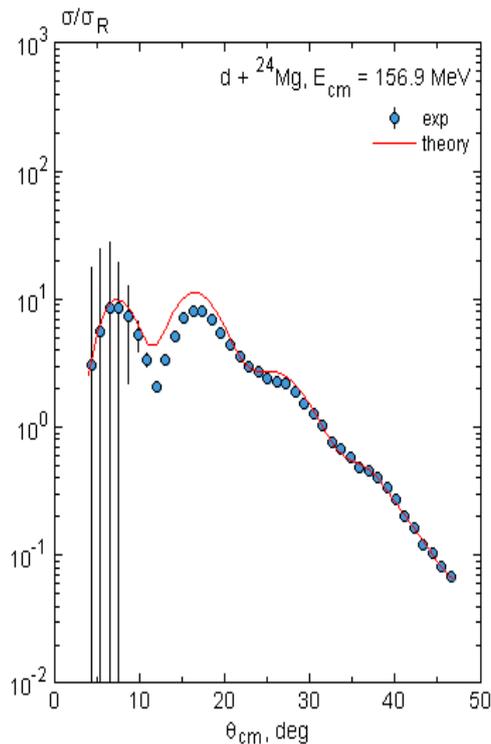


Figure 28: Differential cross-section of  $d + {}^{24}\text{Mg}$  at  $E_{\text{lab}} = 170 \text{ MeV}$ .

The folded potentials of the elastically scattered deuteron from  ${}^{24}\text{Mg}$  at various incident energies were determined. The optical potential components were assumed to be

identical, except for the differing renormalization factors  $N_{r(i)}$ . The renormalization process adjusted the strengths of the folded potential to match experimental findings.

Larger renormalization factors,  $N_r$ , were required for the real component of the optical potential, while smaller values of  $N_i$ , were obtained for the imaginary component of the OP. The radial shape of the strengths of the derived potentials at the various energies were obtained and are shown in Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 & 14. These potentials exhibited strong and attractive qualities of the nuclear force within short internuclear distances of around  $r \sim 6$  fm. Across all studied interactions, the potentials exhibited a smooth decrease over short distances and took the Woods-Saxon shape. Additionally, the differential cross-sections were computed and compared with experimental data in order to assess the effectiveness of the generated optical potential. Plots of the differential cross-sections as a function of angle in the centre of mass ( $\theta_{cm}$ ) are displayed in Figures 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27 & 28. Excellent agreements of differential cross-sections between the experimental data and the current computation were observed at all incident energies. At both small and large angles, the angular distributions showed consistent matches between the computed and experimental values.

## CONCLUSION

Double-folding model calculations of  $d + {}^{24}\text{Mg}$  were performed in the elastic channel within the incident laboratory energies of 60 – 170 MeV. The optical potentials of the density-dependent B3Y-Fetal interaction were derived by convolving it over the ground state densities of the deuteron and  ${}^{24}\text{Mg}$  inside the DFM framework. The derived potentials were attractive and short-ranged over small nuclear distances of about  $0 \leq r \leq 6$  fm. The folded potentials were also deeper at smaller internuclear distances of about  $r \leq 2$  fm. Based on the potential of the double-folding model, the current optical model analysis successfully reproduced the experimental results. This remarkable agreement demonstrated the suitability and reliability of the double-folding model and the B3Y-Fetal interaction as theoretical tools for understanding nuclear reaction data. A consistent explanation of the scattering data was given using the DFM. The results of the current study should motivate interest in using the present formulation of the OP, B3Y-Fetal interaction, as well as its variations to analyse and study nuclear reactions.

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