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ORIGINAL RESEARCH ARTICLE

A Mathematical Model for Water Quality Assessment: Evidence-Based from Selected Boreholes in Federal University Dutse, Nigeria

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ABSTRACT

The present study assessed the quality of water sampled from different boreholes on the campus of Federal University Dutse, Nigeria, using a mathematical modelling approach. A model for estimating water quality was developed based on physicochemical parameters such as pH, electrical conductivity, temperature, turbidity, and total hardness measured from each borehole. The correlation analysis of physicochemical data indicates a strong correlation of about 99% between the real-life data collected from six (6) different boreholes and the model's predictions. From the results, the sensitivity analysis revealed that electrical conductivity plays the highest role in determining water quality, followed by total hardness, temperature has the third highest impact, followed by turbidity, the fourth, and pH has the least impact in determining water quality in the listed boreholes. Therefore, in any case of intervention, the water quality regulatory body should be sent regularly to the tertiary institutions in the state for routine check-ups.

INTRODUCTION

Humans can survive for weeks without eating but cannot go for days without drinking water since water is needed to restore lost fluids through regular physiological processes (Addisie, 2022; Amoo *et al.,* 2021a). Water is a crucial natural resource essential for sustaining life on Earth. However, only around 3% of the total water supply is accessible from groundwater, springs, rivers, and lakes used for water (Elemile *et al.,* 2019; Zieminska & Skrzypski, 2012; Amoo *et al.,* 2018). Furthermore, as no human activity can be accomplished without water, water has an inextricable connection to life. Water is indispensable for life and performs various irreplicable functions (Amoo *et al.,* 2021b). The world's primary source of drinkable water is groundwater, and its chemical composition determines how safe it is for humans, animals, and plants. Because of its impact, groundwater pollution differs from surface water contamination (Amoo *et al.,* 2021b). Natural water source contamination and pollution have become serious challenges in emerging and highly populated nations like Nigeria (Jin *et al.,* 2018). The authors emphasized further that water contamination renders water unfit for ingestion by humans and raises the expense of treatment to achieve acceptable quality. In order to help with the adoption of actions that can be used to avoid water-borne diseases and address the challenges of pollution, water quality evaluation is crucial (Spectrum Laboratories Group, SLG 2021; Adeleye *et al.,* 2022). This assessment helps identify potential water pollutants and ensures that the water quality monitoring meets local and global standards. Since water consumption is unavoidable, water quality evaluation can be considered an important component of environmental monitoring. Temperature and turbidity are examples of the physical characteristics of water, while chemical characteristics include variables like pH and dissolved oxygen.

Several findings on drinking water show that drinking water quality is a global concern, as contaminated and inadequate water sources, coupled with inadequate sanitation practices, contribute to various human diseases (Idris *et al.,* 2019; Siddiqua *et al.,* 2019; Waghmare & Kiwne, 2017; Marusic, 2013). Water bodies are often polluted due to the release of untreated waste, sewage, dissolved oxygen, bacteria, and toxic chemical substances

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from industries. This pollution adversely affects water's physical, chemical, and biological properties. Considering these pressing concerns, this study was conducted to design a mathematical model for water quality assessment based on the parameters of the existing borehole data from Federal University Dutse, Nigeria.

MATERIALS AND METHODS

Formulation of the Model

In this section, we shall address the following subtopics as they unfold.

Basic assumption

(i) Water quality, (W_q) versus Temperature (T): Temperature is one of its most basic properties, and many other parameters depend on temperature for accuracy. With temperature data, we can monitor thermal loading or discharge and determine changes in the thermocline, which affect the health of aquatic species and organisms. Many aquatic organisms are sensitive to high temperatures. Mathematically,

$$\left.\begin{aligned} W_q &\propto T \\ W_q &= W_1 T \end{aligned}\right\} \tag{1}$$

where W_1 is a constant.

(ii) Water quality, (W_q) versus total hardness, (H_t): Total hardness is the sum of the calcium and magnesium concentrations, expressed as calcium carbonate, and measured in milligrams per liter (mg/L). The water hardness based on the concentration of calcium carbonate can be determined using equation (2) as follows:

$$\left.\begin{aligned} W_q &\propto H_t \\ W_q &= W_2 H_t \end{aligned}\right\} \tag{2}$$

where W_2 is a constant.

(iii) Water quality, (W_q) versus the potential of hydrogen (pH): pH measurement is an important parameter in nearly all water quality applications. It is a quantitative measure of the acidity or basicity of aqueous or other liquid solutions. High or low pH values can indicate pollution in environmental sampling and monitoring. The relationship between water quality and pH is given by,

$$\left.\begin{aligned} W_q &\propto P_H \\ W_q &= W_3 P_H \end{aligned}\right\} \tag{3}$$

where W_3 is a constant, and P_H represents pH.

(iv) Water quality, (W_q) versus Electrical Conductivity, (E_C): Electrical conductivity is also an indicator of water quality. Conductivity data is essential in determining the concentration of a solution. Mathematically,

$$\left.\begin{aligned} W_q &\propto E_C \\ W_q &= W_4 E_C \end{aligned}\right\} \tag{4}$$

where W_4 is a constant.

(v) Water quality (W_q) versus Measurement Turbidity, (M_t): Turbidity is the measurement of water clarity. Suspended sediments, such as particles of clay soil and slit, frequently enter the water from disturbed sites and affect water quality. The mathematical model formula for determining water quality based on turbidity is presented by:

$$\left.\begin{aligned} W_q &\propto M_t \\ W_q &= W_5 M_t \end{aligned}\right\} \tag{5}$$

where W_5 is a constant.

First Establishment of Model Parameters Relationship

From our respective postulations above, adding the Equations (1), (2), (3), (4), and (5) gives

$$\left.\begin{aligned} 5W_q &= W_1 T + W_2 H_t + W_3 P_H + W_4 E_C + W_5 M_t \\ W_q &= W_1^1 T + W_2^1 H_t + W_3^1 P_H + W_4^1 E_C + W_5^1 M_t \end{aligned}\right\} \tag{6}$$

where

$$W_1^1 = \frac{W_1}{5}, W_2^1 = \frac{W_2}{5}, W_3^1 = \frac{W_3}{5}, W_4^1 = \frac{W_4}{5}, W_5^1 = \frac{W_5}{5}$$

, T is the temperature, H_t is the total hardness, P_H is the potential of hydrogen, E_C is the electrical conductivity, and M_t is the turbidity measurement.

Nature of the Equation's Basic Assumption

As applied by Ogwumu *et al.* (2018), the nature of the equation view was given to the relationship between the model's variables. Hence, using the parameters, temperature, dissolved oxygen, pH, electrical conductivity, measurement turbidity, and considering Equations (1), (2), (3), (4), and (5), it can also be observed that the relationship between these parameters (T, H_t, P_H, E_C, and M_t) and W_q is a linear equation relationship given by Equations (7) to (11) as follows:

$$W_q = q_1 T + C_1 \tag{7}$$

$$W_q = q_2 H_t + C_2 \tag{8}$$

$$W_q = q_3 P_H + C_3 \tag{9}$$

$$W_q = q_4 E_C + C_4 \quad (10)$$

$$W_q = q_5 M_t + C_5 \quad (11)$$

where C_1, C_2, C_3, C_4 and C_5 are constants.

Second Estimation of the Model Parameter Relationship

By adding our respective Equations (7), (8), (9), (10), and (11), we get Equation (12)

$$W_q = q_1^1 T + q_2^1 H_t + q_3^1 P_H + q_4^1 E_C + q_5^1 M_t + C \quad (12)$$

where C is constant i.e. $C = (C_1 + C_2 + C_3 + C_4 + C_5)/5$, $q_1^1 = \frac{q_1}{5}, q_2^1 = \frac{q_2}{5}, q_3^1 = \frac{q_3}{5}, q_4^1 = \frac{q_4}{5}$, and $q_5^1 = \frac{q_5}{5}$.

General Establishment of the Model Parameter Relationship

From our respective postulations above, if we add Equations (6) and (12) similarly, we have;

$$2W_q = \left(W_1^1 + q_1^1\right)T + \left(W_2^1 + q_2^1\right)H_t + \left(W_3^1 + q_3^1\right)P_H + \left(W_4^1 + q_4^1\right)E_C + \left(W_5^1 + q_5^1\right)M_t + C$$

which gives,

$$W_q = \sigma T_i + \beta H_{ti} + \omega P_{Hi} + \chi E_{Ci} + \delta M_{ti} + \mu \quad (13)$$

where $\sigma = \frac{W_1^1 + q_1^1}{2}, \beta = \frac{W_2^1 + q_2^1}{2}, \omega = \frac{W_3^1 + q_3^1}{2}, \chi = \frac{W_4^1 + q_4^1}{2}, \delta = \frac{W_5^1 + q_5^1}{2}, \mu = \frac{C}{2}$, σ, β, ω, χ, δ and μ are model constants of proportionality.

Analysis of the Model to Evaluate Its Equation Constants

Following a similar procedure reported by Ogwumu *et al.* (2018), we evaluated the constants in the model above by partially differentiating Equation (13) with respect to σ, β, ω, δ and μ, respectively. Then, we solved the resulting Equations using the least squares method. This yielded Equation (14),

$$q_{min} = min\sum(W_q - \sigma T_i - \beta H_{ti} - \omega P_{Hi} - \chi E_{Ci} - \delta M_{ti} - \mu)^2 \quad (14)$$

where: $i = 1, 2, 3\ldots\ldots n$

$$\frac{\partial q}{\partial \sigma} = -2\sum\left(W_q - \sigma T_i - \beta H_{ti} - \omega P_{Hi} - \chi E_{Ci} - \delta M_{ti} - \mu\right)T_i = 0, \quad (15)$$

$$\frac{\partial q}{\partial \beta} = -2\sum\left(W_q - \sigma T_i - \beta H_{ti} - \omega P_{Hi} - \chi E_{Ci} - \delta M_{ti} - \mu\right)H_{ti} = 0, \quad (16)$$

$$\frac{\partial q}{\partial \omega} = -2\sum\left(W_q - \sigma T_i - \beta H_{ti} - \omega P_{Hi} - \chi E_{Ci} - \delta M_{ti} - \mu\right)P_{Hi} = 0, \quad (17)$$

$$\frac{\partial q}{\partial \chi} = -2\sum\left(W_q - \sigma T_i - \beta H_{ti} - \omega P_{Hi} - \chi E_{Ci} - \delta M_{ti} - \mu\right)E_{Ci} = 0, \quad (18)$$

$$\frac{\partial q}{\partial \delta} = -2\sum\left(W_q - \sigma T_i - \beta H_{ti} - \omega P_{Hi} - \chi E_{Ci} - \delta M_{ti} - \mu\right)M_{ti} = 0, \quad (19)$$

$$\frac{\partial q}{\partial \mu} = -2\sum\left(W_q - \sigma T_i - \beta H_{ti} - \omega P_{Hi} - \chi E_{Ci} - \delta M_{ti} - \mu\right) = 0. \quad (20)$$

Hence, from Equation (15), we obtained,

$$\sigma\sum T_i^2+\beta\sum H_{ti}T_i+\omega\sum P_{Hi}T_i+\chi\sum E_{Ci}T_i+\delta\sum M_{ti}T_i+\mu\sum T_i=\sum W_qT_i. \quad (21)$$

Accordingly, from Equation (16), it is obtained that,

$$\therefore\sigma\sum T_iH_t+\beta\sum H_{ti}^2+\omega\sum P_{Hi}H_{ti}+\chi\sum E_{Ci}H_{ti}+\delta\sum M_{ti}H_{ti}+\mu\sum H_{ti}=\sum W_qH_{ti}. \quad (22)$$

Furthermore, from Equation (17), yields

$$\therefore\sigma\sum T_iP_H+\beta\sum H_{ti}P_H+\omega\sum P_{Hi}^2+\chi\sum E_{Ci}P_H+\delta\sum M_{ti}P_H+\mu\sum P_H=\sum W_qP_{Hi}. \quad (23)$$

Additionally, from Equation (18), we have,

$$\therefore\sigma\sum T_iE_{Ci}+\beta\sum H_{ti}E_{Ci}+\omega\sum P_{Hi}E_{Ci}+\chi\sum E_{Ci}^2+\delta\sum M_{ti}E_{Ci}+\mu\sum E_{Ci}=\sum W_qE_{Ci}. \quad (24)$$

Subsequently, from Equation (19), we get,

$$\therefore\sigma\sum T_iM_{ti}+\beta\sum H_{ti}M_{ti}+\omega\sum P_{Hi}M_{ti}+\chi\sum E_{Ci}M_{ti}+\delta\sum M_{ti}^2+\mu\sum M_{ti}=\sum W_qM_{ti}. \quad (25)$$

Also, Solving Equation (20) gives:

$$\therefore\sigma\sum T_i+\beta\sum H_{ti}+\omega\sum P_{Hi}+\chi\sum E_{Ci}+\delta\sum M_{ti}+6\mu=\sum W_{qi}. \quad (26)$$

where i = 1, 2, 3….n and n = 6. However, solving Equations (21), (22), (23), (24), (25), and (26) simultaneously for the values of σ, β, ω, χ, δ and μ.

Research Instrument Used

In an attempt to investigate the extent of water quality, six sampling points (boreholes) designated as BH_1, BH_2, BH_3, BH_4, BH_5, and BH_6 were selected within the university campus. Table 1 presents the borehole sampling location.

Table 1: Borehole sample location with its acronym

Location of borehole	Acronym	Latitude	Longitude
Rukayya Hall	BH_1	0540069	1294281
Aminat Hall	BH_2	0540047	1294185
Shekarau Angyu Hall	BH_3	0540756	1293503
Faculty of science	BH_4	0540506	1294023
Faculty of Agriculture	BH_5	0540812	1294306
Senate building	BH_6	0540627	1294855

Key: BH: borehole

Conversely, the laboratory analysis results were subjected to statistical analysis using Microsoft Excel. The results of the physicochemical attributes of the water samples derived from all the boreholes are presented in Table 2. Furthermore, Tables 3, 4 and 5 present multiplication and summation of the physicochemical data based on Table 2.

Table 2: The result of the physicochemical attributes of the borehole water samples

Sample	Wq	T	pH	M_t	Ec	H_t
BH1	66.00	32.00	7.02	3.43	596	85.5
BH2	70.20	31.00	7.78	0.02	592	85.5
BH3	53.20	31.00	7.80	41.2	690	85.5
BH4	73.20	32.00	7.25	0.01	467	68.4
BH5	69.20	33.00	7.35	0.01	624	102.6
BH6	49.20	32.00	6.53	51.3	422	51.3

$\sum Wq = 381.00$	$\sum T = 191.00$	$\sum pH = 43.73$	$\sum Mt = 95.97$	$\sum E_c = 3391$	$\sum H_t = 478.8$

Table 3: Multiplication and summation of physicochemical data in Table 2.

Wq^2	T^2	pH^2	Mt^2	Ec^2	Ht^2	WqT	WqPh	WqMt
4356	1024	49.2804	11.7649	355216	7310.25	2112	463.32	226.38
4928.04	961	60.5284	0.0004	350464	7310.25	2176.2	546.156	1.404
2830.24	961	60.84	1697.44	476100	7310.25	1649.2	414.96	2191.84
5358.24	1024	52.5625	0.0001	218089	4678.56	2342.4	530.7	0.732
4788.64	1089	54.0225	0.0001	389376	10526.76	2283.6	508.62	0.692
2420.64	1024	42.6409	2631.69	178084	2631.69	1574.4	321.276	2523.96
$\sum Wq^2 = \mathbf{24681.8}$	$\sum T^2 = 6083.00$	$\sum pH^2 = 319.875$	$\sum Mt^2 = 4340.896$	$\sum Ec^2 = 1967329$	$\sum Ht^2 = 39767.76$	$\sum WqT = 12137.8$	$\sum WqPh = 2785.032$	$\sum WqMt = 4945.008$

Table 4: The next multiplication and summation of physicochemical data in Table 2.

W_qE_c	W_qH_t	TP_H	TM_t	TE_c	TH_t	pHM_t	pHE_c	pHH_t	M_tE_c
39336	5643	224.64	109.76	19072	2736	24.0786	4183.92	600.21	2044.28
41558.4	6002.1	241.18	0.62	18352	2650.5	0.1556	4605.76	665.19	11.84
36708	4548.6	241.8	1277.2	21390	2650.5	321.36	5382	666.9	28428
34184.4	5006.88	232	0.32	14944	2188.8	0.0725	3385.75	495.9	4.67
43180.8	7099.92	242.55	0.33	20592	3385.8	0.0735	4586.4	754.11	6.24
20762.4	2523.96	208.96	1641.6	13504	1641.6	334.989	2755.66	334.989	21648.6
$\sum W_qE_c = \mathbf{215730.00}$	$\sum W_qH_t = 30824.46$	$\sum TP_H = 1391.13$	$\sum TM_t = 3029.83$	$\sum TE_c = 107854.00$	$\sum TH_t = 15253.20$	$\sum pHM_t = 680.73$	$\sum pHE_c = 24899.49$	$\sum pHH_t = 3517.30$	$\sum M_tE_c = 52143.63$

Table 5: The final multiplication and summation of physicochemical data in Table 2

M_tH_t	E_CH_t
293.265	50958
1.71	50616
3522.6	58995
0.684	31942.8
1.026	64022.4
2631.69	21648.6
$\sum \boldsymbol{M_tH_t} = \mathbf{6450.98}$	$\sum W_qE_c = 278182.80$

Additionally, the results obtained from comparing the physicochemical properties of the sampled borehole water with the regulated standards are as follows:

$$6083.00\sigma + 15253.20\beta + 1391.13\omega + 107854.00\chi + 3029.83\delta + 191.00\mu = 12138, \tag{27}$$

$$15253.20\sigma + 39767.76\beta + 3517.30\omega + 278182.80\chi + 6450.98\delta + 478.8\mu = 30824.46, \tag{28}$$

$$1391.13\sigma + 3517.30\beta + 319.88\omega + 24899.49\chi + 680.73\delta + 43.73\mu = 2785.03, \tag{29}$$

$$107854.00\sigma + 278182.80\beta + 24899.49\omega + 1967329.00\chi + 52143.63\delta + 3391\mu = 215730, \tag{30}$$

$$3029.83\sigma + 6450.98\beta + 680.73\omega + 52143.63\chi + 4340.9\delta + 95.97\mu = 4945.01, \tag{31}$$

$$191.00\sigma + 478.8\beta + 43.73\omega + 3391\chi + 95.97\delta + 6\mu = 381.00. \tag{32}$$

Hence, solving Equations (27), (28),(29),(30), (31) and (32) simultaneously gives

$$\sigma = 3.477023989$$
$$\beta = -0.278987782$$
$$\omega = 8.117334953$$
$$\chi = 0.008297263$$
$$\delta = -0.43795277$$
$$\mu = -81.76816346$$

Consequently, substituting these values in Equation (13) yields.

$$Wq = 3.477024T - 0.278988H_t + 8.117335P_H + 0.00829726E_C - 0.437953M_t - 81.768163;\ Wq \geq 0 \tag{33}$$

Therefore, Equation (33) is the developed model for estimating the water quality (borehole water).

RESULTS AND DISCUSSION

Numerical results using our emerging model equation constant will be presented in this section. Accordingly, Equation (33) represents our model equation, which has been derived based on the collected data. The collected data were computed to validate and ensure its accuracy, and the results are summarized in Tables 6 and 7. The result of the evaluated correlation coefficient is 0.996544. This indicates the effectiveness of our model equation.

Table 6: Correlation coefficient computation values for the model

Sample	Wq(X)	Wq(Y)	T	pH	M_t	E_c	H_t	$\bar{X}$	$(X-\bar{X})$
BH1	66	66.06981	32	7.02	3.43	596	85.5	63.5	2.5
BH2	70.2	70.22219	31	7.78	0.02	592	85.5	63.5	6.7
BH3	53.2	53.16277	31	7.8	41.2	690	85.5	63.5	-10.3
BH4	73.2	73.13495	32	7.25	0.01	467	68.4	63.5	9.7
BH5	69.2	69.18498	33	7.35	0.01	624	102.6	63.5	5.7
BH6	49.2	49.22517	32	6.53	51.3	422	51.3	63.5	-14.3
	$\sum \text{Wq(X)} = 381$	$\sum \text{Wq(Y)} = 380.9999$	$\sum \text{T} = 191$	$\sum \text{pH} = 43.73$	$\sum M_t = 95.97$	$\sum E_c = 3391$	$\sum H_t = 478.8$	$\sum X = 381$	$\sum X - X = 1.42109$

Table 7: The next correlation coefficient computation values for the model

$\bar{Y}$	$Y-\bar{Y}$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
62.75	3.31981	6.25	11.0211	8.29952725
62.75	7.47219	44.89	55.8337	50.06368707
62.75	-9.5872	106.09	91.9151	98.74851226
62.75	10.3849	94.09	107.847	100.7339714
62.75	6.43498	32.49	41.409	36.67940367
62.75	-13.525	204.49	182.921	193.4050261
$\sum \bar{Y} = \mathbf{376.5}$	$\sum Y-\bar{Y} = \mathbf{4.49987}$	$\sum (X-\bar{X})^2 = 488.3$	$\sum (Y-\bar{Y})^2 = 490.947$	$\sum (X-\bar{X}) = 487.930$

$$r_{x,y} = \frac{\sum_{i=1}^{6}\left(X_i-\bar{X}\right)\left(Y_i-\bar{Y}\right)}{\sqrt{\sum_{i=1}^{6}\left(X_i-\bar{X}\right)^2 \sum_{i=1}^{6}\left(Y_i-\bar{Y}\right)^2}} = \frac{487.930}{\sqrt{(488.3)(490.947)}} = 0.996544.$$

Variable Sensitivity Analysis

Variable sensitivity analysis was carried out for temperature (T), electrical conductivity (E_C), measurement turbidity (M_t), total hardness (H_t), and potential of hydrogen (pH). We do this using the variable sensitivity index equations relation as follows:

$$T_{index} = \frac{\partial W_q}{\partial T} \cdot \frac{T}{W_q}\bigg|_{T=31.8333,\, Ph=7.2883,\, Mt=16.0,\, Ec=565.1667,\; Ht=79.8}$$

$$E_{c-index} = \frac{\partial W_q}{\partial E_c} \cdot \frac{E_c}{W_q}\bigg|_{T=31.8333,\, Ph=7.2883,\, Mt=16.0,\, Ec=565.1667,\; Ht=79.8}$$

$$P_{h-index} = \frac{\partial W_q}{\partial P_h} \cdot \frac{P_h}{W_q}\bigg|_{T=31.8333,\, Ph=7.2883,\, Mt=16.0,\, Ec=565.1667,\; Ht=79.8}$$

$$M_{t-index} = \frac{\partial W_q}{\partial M_t} \cdot \frac{M_t}{W_q}\bigg|_{T=31.8333,\, Ph=7.2883,\, Mt=16.0,\, Ec=565.1667,\; Ht=79.8}$$

$$H_{t-index} = \frac{\partial W_q}{\partial H_t} \cdot \frac{H_t}{W_q}\bigg|_{T=31.8333,\, Ph=7.2883,\, Mt=16.0,\, Ec=565.1667,\; Ht=79.8}$$

The Equations above give the sensitivity; their results are summarized in Table 8.

Table 8: Sensitivity analysis values

S/N	Parameter	Sensitivity Index	Ranking
1	Temperature (T)	1.743145	3rd
2	Electrical conductivity (E_c)	30.947694	1st
3	Potential of hydrogen (pH)	0.399097	5th
4	Measurement turbidity (M_t)	0.876136	4th
5	Total hardness (H_t)	4.369730	2nd

Table 8 shows that the most sensitive parameters, in order of their ranking, are electrical conductivity, total hardness, temperature, measurement turbidity, and pH, respectively. On the other hand, Figure 1 depicts the sensitivity indices of the parameters. As can be observed in Figure 1, electrical conductivity exhibits the highest sensitivity indices, while measurement turbidity exhibits the lowest sensitivity.

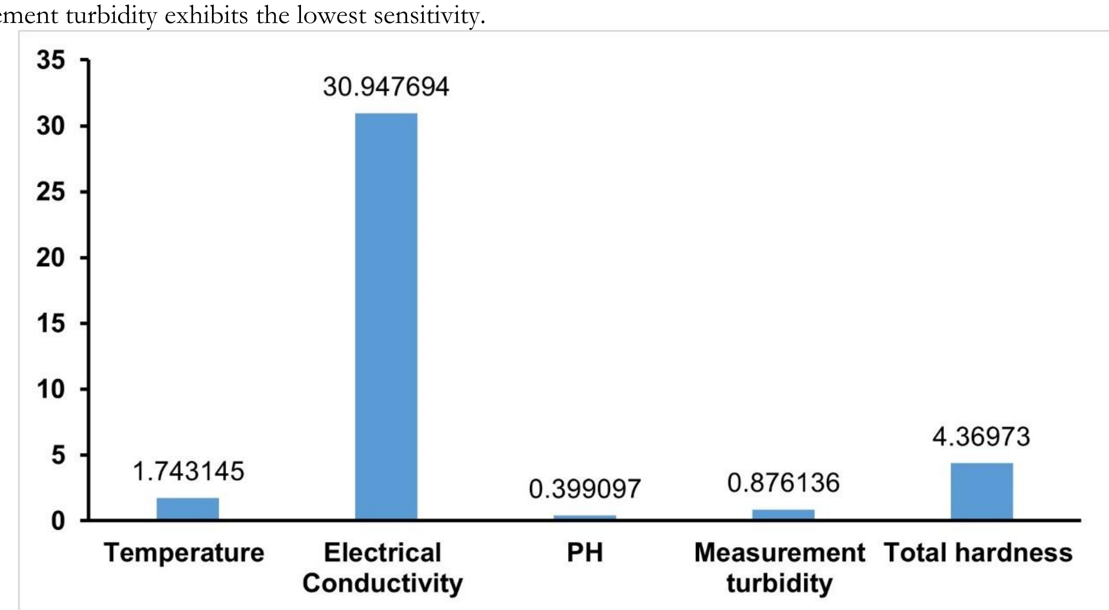

Figure 1: The sensitivity indices of the parameters

CONCLUSION

The study successfully formulated a mathematical model for water quality assessment using physicochemical parameters and applied it to selected boreholes at Federal University Dutse, Nigeria. The analysis of these parameters revealed that variations among different sampling locations and physicochemical factors such as pH, turbidity, total hardness, and electrical conductivity significantly impact water quality. The strong correlation coefficient (r = 0.997) between real-life data (X) and model predictions (Y) confirms the reliability of our water quality model for predictive purposes. Furthermore, the sensitivity analysis table demonstrated that electrical conductivity is the most influential factor in determining water quality, followed by total hardness. Temperature ranks third in impact, while measurement turbidity occupies the fourth position. Overall, pH has the least impact on determining the water quality of the mentioned boreholes. These findings emphasize the importance of considering these parameters for effective water quality assessment and management. It is recommended that the water stored in the storage tanks be cleaned regularly to avert possible physical and microbial contamination. The installation of membrane filters in the water outlets of the tanks should be ensured to effectively filter water meant for human consumption. Finally, relevant stakeholders need to carry out borehole-water purification projects and activities in the Federal University Dutse community geared towards rescuing the users of these boreholes from unsafe health challenges.

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