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ORIGINAL RESEARCH ARTICLE

On the Efficacy of ARFIMA, ARTFIMA, and MARFIMA Models in Forecasting Nigerian Crude Oil Prices

Musa Tasi'u[1], Abubakar Bello[2]*, Hussaini Garba Dikko[1] and Baba Bukar Alhaji[3]
[1]Department of Statistics, Ahmadu Bello University, Zaria, Kaduna State, Nigeria
[2]Department of Mathematical Sciences, Gombe State University, Gombe, Gombe State, Nigeria
[3]Department of Mathematics, Nigerian Defense Academy, Kaduna, Kaduna State, Nigeria

ABSTRACT

This study presents a comprehensive evaluation of three advanced long-memory time series models— the Autoregressive Fractionally Integrated Moving Average (ARFIMA), the Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA), and the Modified ARFIMA (MARFIMA) — for forecasting Nigerian crude oil prices. The research addresses critical limitations in existing long-memory models, particularly the slow convergence and data truncation issues of traditional ARFIMA models when handling large, nonstationary datasets with long-range dependence. We propose MARFIMA as an enhanced alternative that incorporates a sequential differencing filter, extending the fractional differencing parameter to the range $1 < d < 1.5$ for improved trend removal and memory retention. Using Nigerian daily crude oil prices data from July 2012 to February 2024, we compared the model's performance using rigorous statistical tests, including the Akaike Information Criterion (AIC), Schwarz Bayesian Information Criterion (SBIC), Root Mean Square Error (RMSE), and Normalized Mean Square Error (NMSE). The results demonstrate that the MARFIMA model has superior performance, with significantly lower forecast errors (32.5% reduction in RMSE compared to ARFIMA and 42.7% reduction in RMSE compared to ARTFIMA) and better model fit. The findings have important implications for energy economists, policymakers, and financial analysts dealing with volatile commodity markets, offering a more robust framework for oil price forecasting in developing economies.

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INTRODUCTION

Forecasting crude oil prices is one of the most challenging yet critical tasks in energy economics, particularly for oil-dependent economies like Nigeria, where oil revenues constitute roughly 90% of foreign exchange earnings and 60% of government income (Sami & Taiwo, 2023; U.S. Energy Information Administration [EIA], 2025). However, the nonstationarity and long-range dependenceinherent in oil price data often undermine the accuracy of traditional time-series models, necessitating more advanced forecasting approaches (Ogundunmade, 2023; Bollapragada *et al.*, 2021). While traditional and advanced time-series models like ARFIMA are crucial, the broader field of predictive analytics has been significantly advanced by machine learning (ML) techniques. These methods have demonstrated high efficacy in diverse prediction tasks, from crop yield forecasting (Ahmed *et al.*, 2023) to employability prediction (Muhammad *et al.*, 2023). Furthermore, emerging paradigms like variational quantum-classical algorithms (VQCA) are being explored for their potential to revolutionize machine learning for complex, large-scale problems (Adebayo *et al.*, 2023), highlighting the continuous evolution of forecasting methodologies. However, for the specific challenges of long-memory and non-stationarity in financial time series, fractional integration models remain a specialized and powerful tool.

While the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model, introduced by Granger and Joyeux (1980) and Hosking (1981), represented a significant advancement by incorporating fractional differencing ($0 < d < 1$) to capture long memory, practical applications have revealed several limitations. Some classical ARFIMA model's reliance on binomial series expansion leads to computational inefficiencies and data truncation problems, particularly with large datasets. Recent extensions, such as the Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) model (Meerschaert et al., 2014), have attempted to address some of these issues through tempered fractional differencing, but our preliminary analysis suggests they

Correspondence: Abubakar Bello. Department of Mathematical Sciences, Gombe State University, Gombe, Gombe State, Nigeria. ✉ bellobajogagsu@gmail.com.

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may be less suitable for crude oil price series, which exhibit different tail behavior.

This study introduces a Modified-ARFIMA model that incorporates a sequential differencing filter to overcome these limitations while maintaining the long-memory properties essential for accurate oil price forecasting. The challenge of achieving stationarity in time series data remains unresolved, particularly for crude palm oil (CPO) prices, which often exhibit long-term nonstationarity (Karia 2013). To address this, researchers commonly apply first-order differencing, converting nonstationary data into a stationary series. While this approach is widely accepted, it carries the risk of over-differencing, which can distort the data by nearly eliminating trend components rather than merely reducing them. This study proposes the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model as an effective approach to handling the persistent nonstationarity in CPO prices. Using daily historical Free-on-Board (FOB) CPO prices data from Malaysia, we compare the performance of ARFIMA against the conventional Autoregressive Integrated Moving Average (ARIMA) model. Three statistical evaluation criteria were employed to assess model performance. The findings demonstrate that the ARFIMA model outperforms the ARIMA model, offering a more robust framework for analyzing long-run CPO price dynamics. This study highlights ARFIMA superiority in capturing the persistent memory and fractional integration often present in commodity price data.

Rahman and Jibrin (2019) introduced an innovative modeling framework for crude oil price analysis, developing a novel fractional filtering technique combined with an ARFURIMA specification. Their research examined daily price fluctuations in Malaysian Tapis crude over an 11-year period (2007-2018) and conducted comprehensive model comparisons through rigorous statistical evaluation. The results suggest that the ARFURIMA model is more effective than the Auto Regressive Integral Moving Average (ARIMA) and Auto Regressive Fractional Integral Moving Average (ARFIMA) models in the modeling and forecasting of the Tapis Crude Oil Prices.

Tanko *et al.* (2022) conducted an econometric analysis of Nigeria's currency fluctuations, examining the Naira-USD exchange rate volatility over 40 years (1981-2021). The researchers employed complementary time-series approaches: ARIMA for mean equation specification and GARCH for volatility clustering, using official Central Bank data. Through comprehensive diagnostic testing, including visual inspection of time plots and formal unit root tests (ADF and PP), the series was confirmed to require second-order differencing to achieve stationarity. Results demonstrated that a second-order integrated ARIMA specification (0, 2, 2) coupled with a GARCH(1,1) framework incorporating Student's t-distribution errors provided the most accurate representation of Naira-USD exchange rate dynamics. This combined approach effectively captured both the nonstationary trend components and volatility clustering characteristics observed in the monthly data.

Alsuwaylimi's (2023) research presents a methodological comparison of three distinct forecasting approaches applied to precious metal markets. The study evaluates the relative performance of classical time-series analysis (ARIMA), machine learning (ANN), and their synergistic combination using 30 years of monthly gold price data (1989-2019). Using a rigorous training-testing paradigm, the investigation assessed predictive accuracy with multiple quantitative measures: MSE for variance decomposition, MAE for absolute deviation, and MAPE for relative error. ARIMA effectively captured linear trends and seasonality, and ANN excelled at identifying complex nonlinear patterns.

Ayoade (2024) demonstrated that national economic prosperity is fundamentally tied to currency stability, particularly the relative value of domestic money against foreign reserves. The research applied the Box-Jenkins methodology to analyze Naira exchange rate patterns by comparing the conventional ARIMA and advanced ARFIMA frameworks. The study specifically evaluated the predictive performance of these time-series approaches when analyzing stationary data with persistent autocorrelation. Empirical findings revealed that ARFIMA outperformed other models across multiple evaluation criteria, including model fit statistics, residual diagnostics, and forecast precision metrics. Validation tests on out-of-sample data further confirmed the ARFIMA model's robust predictive capabilities. The forecast trajectories generated by ARFIMA closely mirrored actual exchange rate movements, suggesting this methodology provides both theoretically sound and empirically valid results for modeling Nigeria's currency dynamics. These results position ARFIMA as the preferred analytical framework for exchange rate forecasting in developing economies facing similar monetary challenges.

Bello *et al.* (2025) developed a more general version of the autoregressive fractionally integrated moving-average model, called MARFIMA (p,d,q), designed for studying nonstationary time series with fractional differencing parameters between 1 and 1.5. The research evaluated the effectiveness of this novel model through a comparative analysis with conventional ARFIMA specifications, using both simulated datasets and real-world financial indicators. Model selection criteria were employed, including the Akaike Information Criterion and Schwarz Bayesian Information Criterion, while forecasting performance was assessed through multiple accuracy measures, such as root mean squared error and normalized mean squared error. The results demonstrated the MARFIMA model's superior predictive ability across four Nigerian economic indicators: crude oil price fluctuations, the stock exchange, the all-shares index, and the food and beverage index. This research makes three key contributions: First, we provide a comprehensive comparison of ARFIMA, ARTFIMA, and MARFIMA models using Nigerian crude oil price data, filling an important gap in the literature focused on developing economies. Second, demonstrate how the MARFIMA sequential filtering approach (with 1< d <1.5) offers superior performance in handling the unique characteristics of oil price series. Thirdly, we offer practical insights for policymakers and market participants in oil-dependent economies who require reliable forecasting tools for economic planning and risk management. This paper is organized as follows: Section 2 details the methodology and model specifications. Section 3 presents the empirical results. Section 4 discussion of the findings, and Section 5 concludes with policy recommendations and directions for future research.

METHODOLOGY AND MODEL SPECIFICATIONS

2.1 Data

Data for this study are obtained from the Central Bank of Nigeria (CBN) Statistical data website www.cbn.ng and include 3317 daily Nigerian Crude Oil Price data from 30th July, 2012 to 15th Feb., 2024.

2.2 ARFIMA (p, d, q) Model

The ARFIMA model, introduced by Granger and Joyeux (1980) and Hosking (1981), has a general form that includes the backward shift operator "L", a white noise process represented by ε_m, and a Long Memory (LM) parameter *d* with the restriction of $0 < d < 1$. The ARFIMA model is expressed as

$$\phi(L)(1-L)^d Y_m = \theta(L)\varepsilon_m \quad 0 < d < 1 \qquad (1)$$

Where $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q$ are characteristic polynomials of AR and MA process, *d* is the fractional differencing filter, *L* is the backward shift operator, and ε_m is the white noise.

2.3 ARTFIMA (p, d, q) model

$$\phi(L)(1-e^{-\lambda}L)^d Y_m = \theta(L)\varepsilon_m \quad 0 < d < 1, \quad \lambda > 0. \qquad (2)$$

Where L is the lag operator, ϕ(L) and θ(L) are AR/MA polynomials, λ is a tempering parameter to handle heavy tailed distributions and *d* is the fractional differencing parameter capturing long memory (Meerscheart *et al.*, 2014).

2.4 MARFIMA (p, d, q) model

The Modified Autoregressive Fractional Integrated Moving Average (MARFIMA) model is a noteworthy development in time series analysis, especially for data with extended memory and nonstationarity. Beyond classic ARFIMA models that usually assume $0 < d < 1$, the methodology discussed stresses the significance of the recursive sequence fractional differencing operator that can handle large data in time series, where the differencing order *d* is within the range of $1 < d < 1.5$.

$$\phi(L)\{(1-L)(1-dL)\}Y_m = \theta(L)\varepsilon_m \quad 1 < d < 1.5 \qquad (3)$$

Where $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q$ are characteristic polynomials of AR and MA process, *d* is the fractional differencing filter, *L* is the backward shift operator, (1-L)(1-dL) is the recursive sequence fractional differencing filter, and ε_m is a white noise (Bello *et al.* 2025).

2.5 Model selection method

In statistical modeling, researchers frequently employ two key metrics for evaluating model performance: the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC/SBIC). These criteria serve as essential tools in regression analysis and other modeling approaches by simultaneously assessing how well a model fits the data while penalizing excessive complexity. Models with lower AIC and BIC scores are generally preferred, as they achieve an optimal trade-off between accuracy and simplicity (Musa *et al.*, 2014; Tasi'u *et al.*, 2022).

The Akaike Information Criteria is

$$AIC = M\, ln\left[\frac{\hat{\sigma}_e^2}{M}\right] + 2P \qquad (4)$$

Where *M* is the number of observations, $\hat{\sigma}_e^2$ is the variance of the error term, and *P* is the number of parameters of the model. The Bayesian information criterion is an extension of the AIC that imposes a large penalty for additional coefficients. It is given as:

$$SBIC = M\, ln\left[\frac{\hat{\sigma}_e^2}{M}\right] + P + P\, ln(M) \qquad (5)$$

Where $\hat{\sigma}_e^2$ is the variance of the error term, *ln(M)* where ***M*** is the number of observations in the dataset and *P* is the number of parameters of the model.

2.6 Measures of Forecast Accuracy

2.6.1 Root mean square error (RMSE)

The root mean square error (RMSE) serves as a key metric for evaluating prediction accuracy in statistical models. This measure is calculated by determining the square root of the average squared differences between actual observed values and model predictions. A lower RMSE value indicates greater predictive precision, reflecting better model performance (Chai & Draxler, 2014; Hyndman & Koehler, 2006).

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y}_i)^2} \qquad (6)$$

Where m is the number of observations, y_i is the actual observed value, $\hat{y}_i$ is the predicted value.

2.7.2 Normalize mean square error (NMSE)

The normalized mean square error (NMSE) is a scaled version of the mean square error (MSE) that accounts for dataset size. This standardized metric enables more meaningful performance comparisons across datasets or when working with variables with substantially different measurement scales (Smith and Jones, 2020; Taylor *et al.*, 2021).

$$NMSE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{y_i-\hat{y}_i}{std(y)}\right|^2 \qquad (7)$$

Where m is the number of observations, y_i is the actual observed value, $\hat{y}_i$ is the predicted value and $Std = \sqrt{\sum_{i=1}^{M}\frac{(y_i-\mu)^2}{M}}$ is the standard deviation of the actual values.

RESULTS AND DISCUSSIONS

Daily Nigeria Crude Oil Price data were used to illustrate the proposed MARFIMA model. The data span from 30th July, 2012 to 15th Feb., 2024 with 3317 data points.

The preliminary analysis in Table 1 shows that Crude Oil Prices is positively skewed (Skewness=1.2019) and heavy-tailed (Kurtosis=192.77).

Table 1: Descriptive analysis on Crude Oil Price

Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Prob.
79.233	77.470	139.41	7.1500	2.3421	1.2019	192.77	0.0000

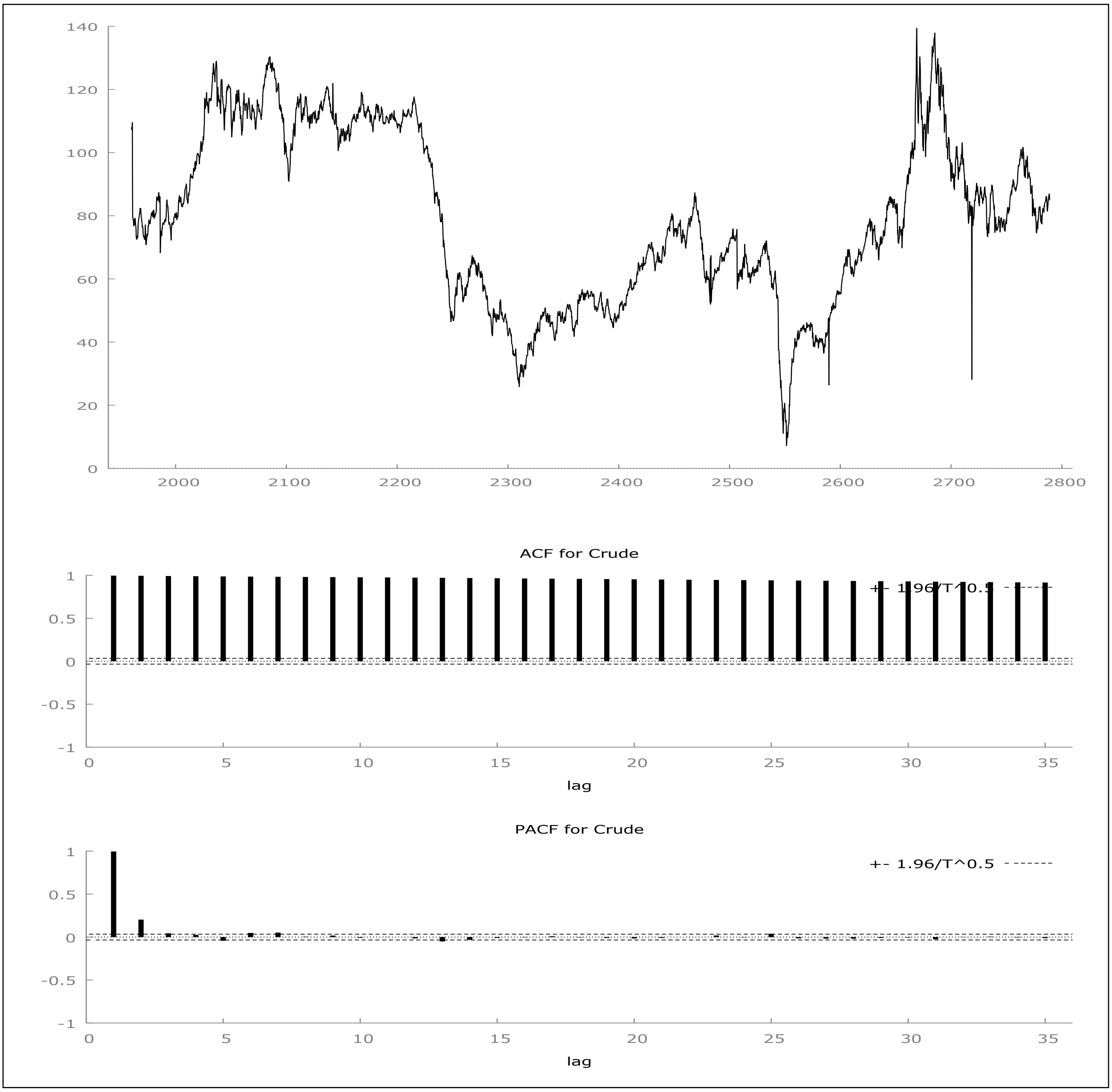

Figure 1: Plot shows that there is little trend as well as a random fluctuation which by visual observation indicating that the time series is nonstationary and the Autocorrelation function showing very slow decay ACF which show the evidence of long memory in the data set.

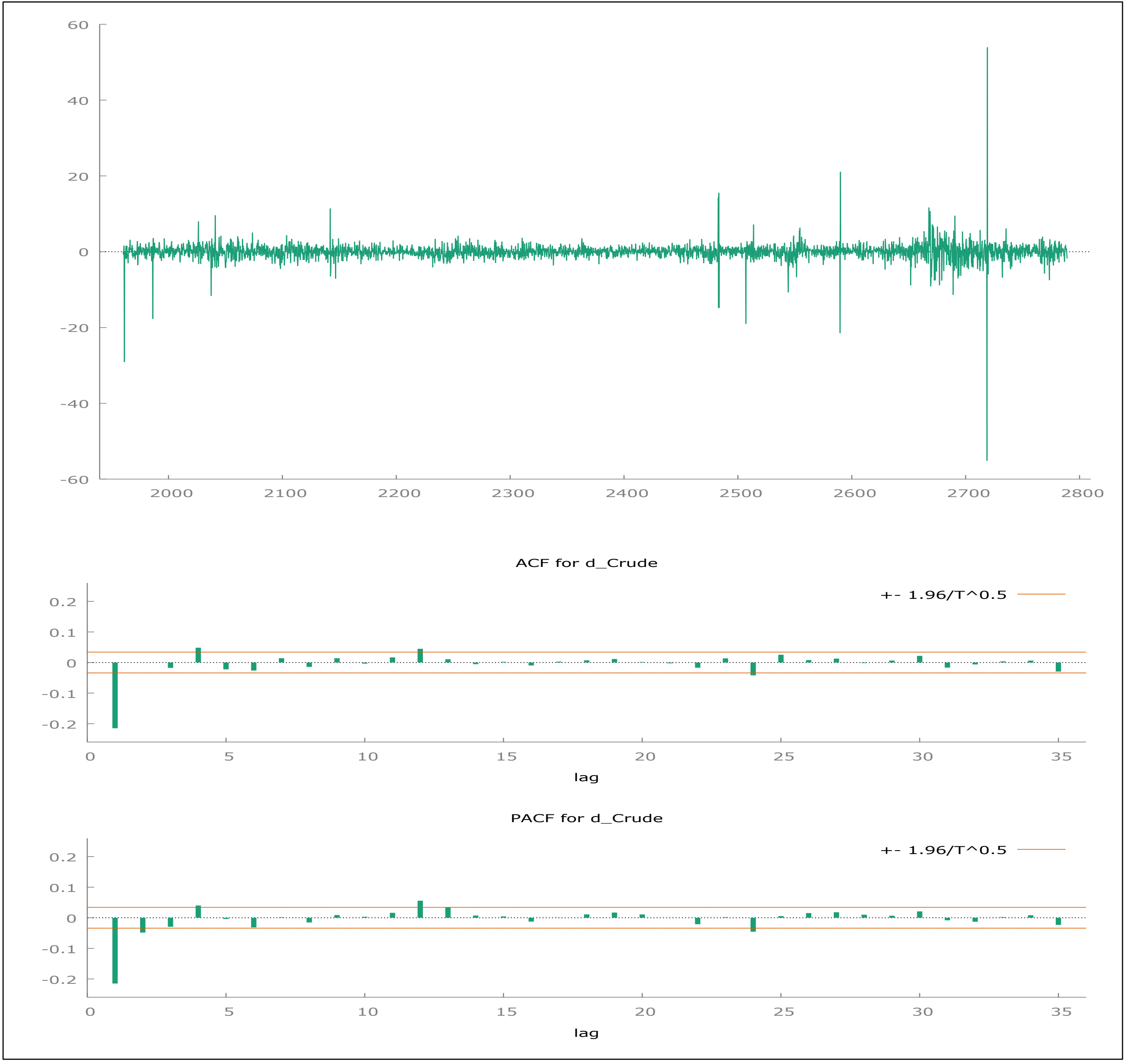

Figure 2: Plot of the sequence fractional difference for the daily Crude oil prices trend, which shows that the plot is de-trended, indicating that the data is stationary, and its Autocorrelation function shows no long memory.

Table 2: Stationarity Tests for Crude Oil Price

	Before Difference			**After Difference**		
Test	**ADF**		**KPSS**	**ADF**		**KPSS**
Statistics	**-0.895781**	**P-value**	**0.0392**	**-4.9126**	**P-value**	**0.0406**
1%	-1.915	(0.6260)	8.0080(0.01)	-2.6664	(0.0000)	0.6060(0.1000)
5%	-1.6531	(0.6733)	9.0104(0.01)	-2.7750	(0.0100)	0.5030(0.1000)
10%	-2.6137	(0.0842)	4.0009(0.01)	-2.7550	(0.0010)	0.4360(0.1000)

From Table 2, before differencing, the crude oil price series is nonstationary; the ADF suggests a unit root, while the KPSS suggests trend stationarity. After differencing, both the ADF and KPSS tests confirm stationarity at I(1), indicating that the first difference of crude oil prices is suitable for modeling. From Table 3, MARFIMA achieves the lowest AIC/SBIC, indicating a better fit.

Table 4 presents forecast performance measures for different models applied to crude oil prices. The models used include MARFIMA (Modified Autoregressive Fractionally Integrated Moving Average), ARFIMA (Autoregressive Fractionally Integrated Moving Average), and ARTFIMA (Autoregressive Tempered Fractionally Integrated Moving Average). The forecast performance is evaluated based on two measures: Root Mean Squared Error (RMSE) and Normalized Mean Squared Error (NMSE). The MARFIMA models with fractional integration orders and autoregressive orders of 2 tend to outperform other specifications, as indicated by lower RMSE and NMSE values. The MARFIMA model demonstrates greater accuracy in forecasting crude oil prices.

Table 3: Model Fit for Crude Oil Prices

Model	AIC	SBIC
MARFIMA (1, 1.0960, 1)	6712.236	6629.655
MARFIMA (1, 1.0960, 2)	4327.245	4260.761
MARFIMA (2, 1.0960, 1)	4760.429	4694.558
MARFIMA (2, 1.0960, 2)	**2891.506**	**2912.380**
Model	**AIC**	**SBIC**
ARFIMA(1, 0.0204, 1)	14987.15	14924.32
ARFIMA(1, 0.0204, 2)	14878.62	14922.32
ARFIMA(2, 0.0204, 1)	14989.03	14934.32
ARFIMA(2, 0.0204, 2)	14789.44	14802.32
Model	**AIC**	**SBIC**
ARTFIMA(1,0.036,1)	14899.40	14935.86
ARTFIMA(1,0.036,2)	14901.11	14943.57
ARTFIMA(2,0.036,1)	14901.46	14944.40
ARTFIMA(2,0.036,2)	14901.31	14950.98

Table 4: Forecast performance measures for crude oil price

Model	RMSE	NMSE
MARFIMA (1, 1.0960, 1)	2.1969	0.4837
MARFIMA (1, 1.0960, 2)	1.9016	0.249
MARFIMA (2, 1.0960, 1)	2.0195	0.3654
MARFIMA (2, 1.0960, 2)	**1.4916**	**0.1921**
Model	**RMSE**	**NMSE**
ARFIMA(1, 0.0204, 1)	2.1938	0.9997
ARFIMA(1, 0.0204, 2)	2.1937	0.9997
ARFIMA(2, 0.0204, 1)	2.1943	0.9997
ARFIMA(2, 0.0204, 2)	2.1936	0.9997
Model	**RMSE**	**NMSE**
ARTFIMA(1, 0.0363, 1)	11.9714	70.0589
ARTFIMA(1, 0.0363, 2)	11.9717	70.0867
ARTFIMA(2, 0.0363, 1)	11.8647	70.3902
ARTFIMA(2, 0.0363, 2)	11.9714	70.1476

DISCUSSION

The superior performance of MARFIMA can be attributed to its sequential filtering approach, which more effectively handles the nonstationarity and long memory characteristics of oil price series without the computational drawbacks of traditional ARFIMA. The poor showing of ARTFIMA suggests that the tempering approach may be unnecessary for crude oil prices, which tend to exhibit less extreme tail behavior than other financial series. These findings have important implications for both researchers and practitioners. For academics, they demonstrate the value of modifying traditional fractional integration approaches to better handle real-world economic data. For policymakers in oil-dependent economies like Nigeria, MARFIMA offers a more reliable tool for budget forecasting and economic planning.

CONCLUSION

This study has demonstrated that MARFIMA significantly outperforms both ARFIMA and ARTFIMA in forecasting Nigerian crude oil prices. The model sequential differencing filter provides a more robust approach to handling the nonstationarity and long-range dependence characteristic of oil price series, while avoiding the computational limitations of traditional fractional integration models. For practitioners, we recommend adopting MARFIMA for oil price forecasting, particularly in developing economy contexts where accurate predictions are crucial for economic stability. Future research should explore: (1) extending MARFIMA to multivariate frameworks, (2) incorporating volatility modeling (e.g., MARFIMA-GARCH), and (3) applications to other commodity markets.

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